An Introduction to Rasch Analysis

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Outlines

- Introduction
- Rasch Analysis Categories
- Rasch Analysis in R

Learning outcomes

- Understand the basic concepts in Rasch analysis
- Understand the results of Rasch analysis for dichotomous items

Introduction

What is Rasch Model

- George Rasch's Measurement Model in 1960
- A probabilistic model for intelligence and attainment tests
- Probability of correct answer depends on TWO facets: Person Ability and Item Difficulty (Facility)

What is Rasch Model

- *P*(Correct) ← Person Ability + Item Difficulty (Facility)
- ↑ Ability + ↓ Difficulty → ↑ Pr(Correct)
- ↓ Ability + ↑ Difficulty → ↓ Pr(Correct)

What is Rasch Model

- Map Ability and Difficulty on the same standardized scale using Logit Transformation
- Proportion (p) of correct → Log Odds

$$\text{Odds} = \frac{p}{(1-p)}$$

$$\log \text{Odds} = \log(\text{Odds})$$

What is Rasch Model

- Ability of Person *n* is

$$B_n = \log\left[\frac{p_n}{1-p_n}\right]$$

- Difficulty of Item *i* is

$$D_i = \log\left[\frac{p_i}{1-p_i}\right]$$

What is Rasch Model

- Probability of correct (X) for Person n and Item i based on Rasch model is

$$P\left(X_{ni} \mid B_n, D_i\right) = \frac{\exp\left(B_n - D_i\right)}{1 + \exp\left(B_n - D_i\right)}$$

Practical

- Let's calculate all these in Excel
- data_10.xls > Ability & Difficulty (original data from Dr. Nurhanis)

Rasch Analysis Categories

Analysis Categories

Three categories of Rasch analysis:

- Calibration
- Model-data fit
- Other validity evidence

Calibration

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Fit Rasch model to estimate:

- Each Item **Difficulty**
- Each Person **Ability**

Range:

-ve → zero → +ve
Easier → Middle → Difficult
Weak → Middle → Bright

NO Discrimination – not part of Rasch model, assumed to be the same and constant (= 1)

Model-data fit

Three categories of Rasch analysis activities:

- Calibration
- Model-data fit
- Other validity evidence

Before calibration:
Dimensionality assessment – unidimensionality (one dimension / trait)
- Factor analysis for categorical data
- EFA on tetrachoric correlations
- CFA using estimation methods that handle categorical data

After calibration:
Item & Person Fits
- INFIT – weighted fit statistics
- OUTFIT – unweighted fit statistics
- SEPARATION RELIABILITY – like Cronbach's alpha

Unidimensionality
Graphical assessment

Model-data fit

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Around 1 for raw /
Around 0 for z/t (±2)

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Responses NEAR an item's difficulty closely match what are expected by the model

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Responses AWAY from an item's difficulty closely match what are expected by the model

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Same as alpha > 0.7
Consistency

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How well an instrument can separate items in terms of their latent variable difficulties

Same as alpha > 0.7
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- % variance explained
- PCA of standardized residuals

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- Maps – Wright's & Pathway
- Item characteristic curve (ICC)

Other validity evidence

Three categories of Rasch analysis activities:

- Calibration
- Model-data fit
- Other validity evidence

- Invariance of item parameters
- Differential item functioning (DIF)
- Other typical construct validity evidence

Other validity evidence

Three categories of Rasch analysis activities:

- Calibration
- Model-data fit
- Other validity evidence

- Split sample into two-halves randomly
- Fit Rasch model
- Correlate between two sample estimates

- Invariance of item parameters
- Differential item functioning (DIF)
- Other typical construct validity evidence

Other validity evidence

Three categories of Rasch analysis activities:

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- Whether performance on any of the items differs for certain groups (e.g. male vs female)
- Probability of correctly responding to an item should be the same for males and females

- Invariance of item parameters
- Differential item functioning (DIF)
- Other typical construct validity evidence

Other validity evidence

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Comparison vs known criteria, other instruments/variables

- Invariance of item parameters
- Differential item functioning (DIF)
- Other typical construct validity evidence

Rasch Analysis in R

Practical

- Let's obtain all these in R
- data_10.xls (original data from Dr. Nurhanis)
- practical_rasch.html (tutorial in R)

References

- Bond, T. G., Yan, Z., & Heene, M. (2021). *Applying the Rasch model: Fundamental measurement in the human sciences* (4th ed.). Rouledge.
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